

Chapter notes: 5 The theory of functions

Overview

This is quite a theoretical chapter. We recommend that functions and ideas from chapters 2 and 3 are taught first, to allow sufficient breadth of functions to be used. It requires approximately four teaching hours.

Introductory problem

This problem is really about domains. It works for any number except zero. The worked solution is given at the end of the chapter, page 150; the idea being that students should be able to answer the question using the methods covered in the chapter.

5A Relations, functions and graphs, p118

Although the concept of a relation is not explicitly part of the IB syllabus, it is difficult to explain functions without reference to relations.

5B Function notation, p123

Students who find this topic relatively easy may find the Extension worksheet on the CD-ROM, including questions on functional equations, of interest.

5C Domain and range, p125

The proof of the difficulty of setting $\frac{1}{0} = \infty$ (as described in the ‘Research explorer’ box on page 126)

is that we also would have to set $\frac{2}{0} = \infty$ so that:

$$\frac{1}{0} = \frac{2}{0}$$

$$\Rightarrow 1 = 2$$

Some academics have indeed suggested that $\frac{1}{0}$ should be accepted, but this requires that division is no longer the inverse operation of multiplication. See the link below for more information:

http://www.bbc.co.uk/berkshire/content/articles/2006/12/06/divide_zero_feature.shtml

The concept of continuity is new in the 2012 syllabus review.

The issue explored in the ‘Research explorer’ box (page 129) explores the difficulties with raising negative numbers to rational powers. That is, is $(-2)^1$ the same as $((-2)^{0.5})^2$?

Hints for the grade 7 questions:

9. The difficulty here is in avoiding multiplying $\frac{8x-4}{x-12} > 0$ by $x-12$.

10. Consider the domain without the restriction.

5D Composite functions, p131

Hints for the grade 7 questions:

8. The range of the inner function must be the domain of the outer function.

9. Try $fgg^{-1}(x)$.

5E Inverse functions, p133

Key point 5.8 is a slight simplification, as it is only true if x is in the domain of both $f(x)$ and $f^{-1}(x)$.

For example, $\sqrt{x^2} = x$ only if $x \geq 0$. However, this is not required for the IB syllabus.

Hints for the grade 7 questions:

15. Set $gg(x) \equiv x$, multiply out the fractions and compare coefficients.

5F Rational functions, p141

The 'Research explorer' box on page 143 refers to the prime number theorem. This says that the number of prime numbers less than or equal to n is approximately equal to $\frac{n}{\ln n}$, with the approximation getting better and better for large values of n .

The 'Theory of knowledge issues' box on p145 is intended to get the students to think about what mathematical knowledge is convention and what is needed for consistency.